Dispersion with Pulsating Turbulent Flow

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Studies of unsteady turbulent flow dispersion in the literature are few in number. For a solute in pulsating laminar flow through a circular tube, Aris (*1*), using the method of moments, found that the dispersion coefficient contains terms proportional to the square of the amplitude of the pressure pulsation. From this result he concluded that the effect of pulsating flow on dispersion is unimportant unless the amplitude of the pressure gradient fluctuations is larger than the mean pressure gradient.

Taylor and Leonard (*13*) in two experiments, one with steady turbulent flow and the other with pulsating flow at the same mean Reynolds number, found that with only a 7.3% variation in the amplitude of the mean velocity, the effective axial diffusion coefficient for unsteady flow was about eleven times greater than the effective axial diffusion coefficient for steady flow. However, when this investigation was continued by Taylor (*12*), the earlier result was not reproduced. Although the newer data (*12*) are too scattered to indicate any definite trend as to how the unsteady flow dispersion coefficient depends on velocity amplitude and frequency, they do indicate that the effects are small.

The purposes of this note are to report results of:

1. Solving the convective diffusion equation with pulsating turbulent flow and steady turbulent flow for the dispersion coefficient in a straight smooth circular tube.
2. Comparing the steady flow and pulsating flow dispersion coefficients with each other and with existing experimental data to determine if the effects of pulsation on the dispersion process can be predicted by the dispersion model and a phenomenological expression for the eddy diffusivity.

This study was motivated by the original experimental results of Taylor and Leonard (*13*). It was not until the study was nearly complete that the later, conflicting data (*12*) were found.

ANALYSIS

The crucial point in the mathematical treatment of the pulsating flow problem is the selection of an expression for the eddy diffusivity. To the authors' knowledge, there are no existing experimental results from which one might obtain an expression for the unsteady-flow turbulent viscosity coefficient. There are, however, indications in the literature that a quasi-steady approach can yield useful information about pulsating flows.

For example, Brown et al. (*3*) were able to predict experimental pressure attenuation factors in tubes with small amplitude disturbances superimposed on a gross turbulent flow using a time invariant eddy viscosity. The lowest frequency studied experimentally was 50 cps. The experiments reported by Schultz-Grunow (*9*) covered the low frequency range with large amplitude fluctuations, some of which were greater than the mean flow velocity. His results showed that the steady flow friction factor was useful in predicting the instantaneous pressure drop.

While these are macroscopic results and do not yield information pertaining to the structure of pulsating turbulent flow, they do provide a basis for assuming a quasi-steady model; namely, it will be assumed that the eddy diffusivity is time independent and can be evaluated from available expressions at the mean Reynolds number for the flow. This is clearly a reasonable assumption for the low frequency range studied here and has been used by Lienhard and Tien (*7*) in analyzing unsteady turbulent channel flow. Its validity at the higher amplitudes employed here is still open to question.

From the available expression for the eddy diffusivity, we choose Cess' (*2, 8*) for the following reasons:

1. A single, continuous function expresses the eddy diffusivity distribution from the center of the tube to the wall which is convenient for numerical calculations.
2. Blanco (*2*) has shown that the Cess expression follows experimental trends better than the expressions of Deissler, Gill, Ranni, Sleicher, and Wasan.

When Taylor's dispersion model (*10, 11*) and the series method developed by Gill (*5*) are used to solve the convective diffusion equation for a step change in concentration at $z = 0$, it is possible to divide the unsteady flow dispersion coefficient into two parts: a steady state coefficient which corresponds to the steady flow case and the unsteady part of the dispersion coefficient which is effected by the pulsations. The mathematical details for the calculation of the dispersion coefficients are given in reference (*8*). In the analysis, a turbulent Schmidt number of unity was assumed and a sinusoidal pressure gradient was used

$$\frac{dP}{dz} = G_s + G_u \sin 2\pi m'\tau$$

Dispersion coefficients have been calculated over the following ranges of parameters: $5{,}000 \leq N_{Re_s} \leq 30{,}000$, $100 \leq N_{Sc} \leq 1{,}000$, 0.0 cps $\leq m \leq 0.618$ cps. and $0.0 \leq G_u/G_s \leq 0.6$.

RESULTS AND DISCUSSION

The results of observations of steady flow axial dispersion in turbulent flow have been reported by a number of investigators. In agreement with Tichacek et al. (*14*), we found that the calculated value of the steady dispersion coefficient is very sensitive to the expression used to describe the turbulent velocity profile. Several results for the dimensionless dispersion coefficient k_s/du_o versus N_{Re_s} are plotted on Figure 1. It is interesting to note that H. M. Taylor's (*12*) experimental data for the large pipe diameter, 0.745 in. I.D., agree well with the Tichacek et al. (*14*) analysis (this was noted also by H. M. Taylor), and the small pipe diameter, 0.242 in. I.D., data agree well with the present analysis.

The instantaneous unsteady flow dispersion coefficient is plotted vs. $\theta - \theta_o$ in Figure 2, where θ equals to 2π times the product of the dimensionless frequency and the di-

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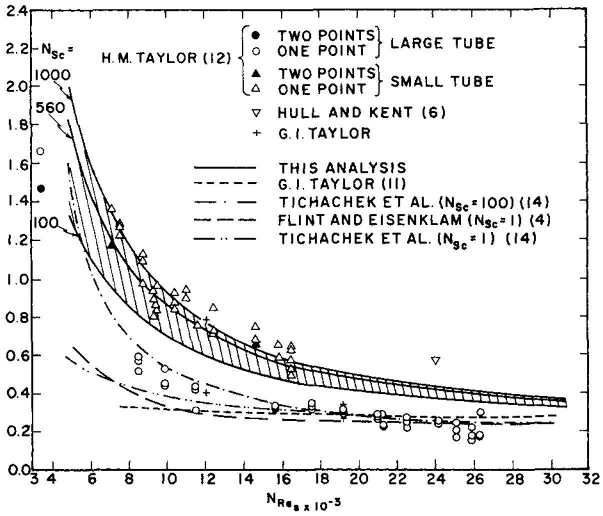

Fig. 1. Comparison of theoretical and experimental dispersion coefficients in steady turbulent flow.

mensionless time, and θ_o is the period over which transients decay completely. It can be seen that pulsating flow has a significant effect on the instantaneous dispersion coefficient, particularly at the larger values of G_u/G_s. The time averaged unsteady flow dispersion coefficient is taken over one period of pulsation, which is the length of the scale shown in Figure 2. There is no noticeable trend in either H. M. Taylor's data or our analytical results for the time averaged unsteady flow dispersion coefficient to indicate a Reynolds number dependency other than that obtained for steady flow.

Because the results are relatively independent of the Schmidt number, and because the Schmidt number for *Azo* Scarlet *Y* in water used by Taylor is unknown, we have compared our results for $N_{Sc} = 560$ with Taylor's data in Figure 3. Here the dimensionless time averaged dispersion coefficient for $N_{Re_s} = 8730$ and $m = 0.412$ cps. is plotted vs G_u/G_s to determine the effect of amplitude of the pressure gradient on the dispersion coefficient. As the velocity amplitude appears as a parameter in the experiments of Taylor and the amplitude ratio G_u/G_s is used here, the inset on Figure 3 gives the peak-to-peak mean velocity vs. G_u/G_s for $N_{Re_s} = 8730$ and $m = 0.412$ cps. as reference. This relationship was calculated using the unsteady flow velocity results (*8*). Excluding the data for small values of G_u/G_s, both the present analysis and the previous experimental data indicate that the dispersion coefficient can be increased by increasing the pulsation amplitude.

A comparison of the ratio of time averaged unsteady dispersion coefficients over the range of parameters considered here shows that $\overline{k}$ exceeds k_s by only 5% or less if G_u/G_s is less than 0.4. At higher amplitudes where the quasi-steady assumption is more tenuous, it is found that unless the amplitude of the fluctuations in the pressure gradient is greater than about one half of the mean pressure gradient, the effect of periodic flow on the dispersion coefficient is less than 10%.

The present results show that the pulsing frequency has a negligible effect on the dispersion coefficient for the range of parameters studied except that the time averaged unsteady flow dispersion coefficient decreases slightly with increasing frequency. This phenomenon may be due to two mechanisms:

1. Increasing pulsing frequency enhances eddy formation, which would lead to an increase in transverse mixing and a decrease in axial mixing, and
2. Increasing pulsing frequency decreases the velocity amplitude, and thus, decreases the dispersion coefficient (*8*).

CONCLUSIONS

The major conclusions drawn from this investigation are as follows:

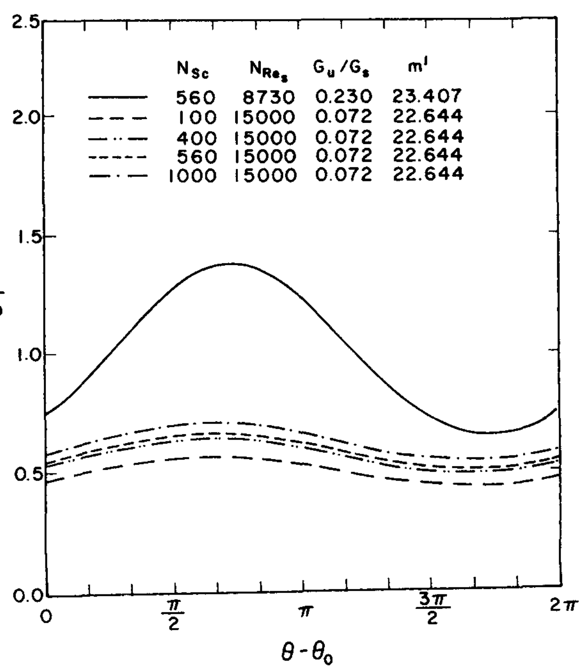

Fig. 2. k/du_0 vs. θ-θ_0 for several sets of parameters.

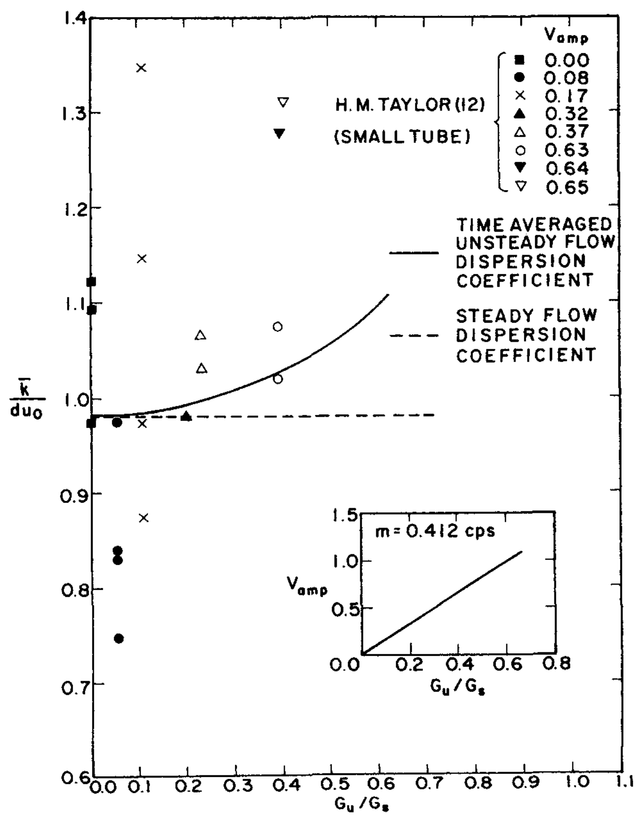

Fig. 3. k/du_0 vs. G_u/G_s for $N_{Re_s} = 8{,}730$, $N_{Sc} = 560$, and $m' = 23.407$ ($m = 0.412$ cps.).

1. Although flow pulsations can generate significant changes in the instantaneous dispersion coefficient, they do not greatly increase axial mixing. That is, unless the amplitude of the fluctuations in the pressure gradient is greater than about one half of the mean pressure gradient, the time averaged dispersion coefficient is not more than 10% greater than the steady flow dispersion coefficient.

2. Changing the frequency of pulsation does not affect axial dispersion as much as changing the pulsation amplitude, and the time averaged unsteady flow dispersion coefficients increase slightly with decreasing frequency.

ACKNOWLEDGMENT

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NOTATION

d = diameter
G_s = dimensionless steady component of the pressure gradient
G_u = dimensionless amplitude of the unsteady component of pressure gradient
k = dimensional unsteady flow dispersion coefficient
k_s = dimensional steady flow dispersion coefficient
$\bar{k}$ = the time averaged unsteady flow dispersion coefficient
m = frequency
m' = dimensionless frequency (mR^2/ν)
p = dimensionless pressure
R = radius
N_{Re_s} = steady flow Reynolds number or time mean Reynolds number for unsteady flow
N_{Sc} = Schmidt number
u_0 = steady flow mean velocity or time mean bulk velocity
V_{amp} = velocity amplitude
z = dimensionless axial coordinate
θ = $2\pi m'\tau$
ν = kinematic viscosity
τ = dimensionless time

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Investigation into Reduced Property Correlations for Aliphatic and Aromatic Aldehydes

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The effect of pressure on the viscosity of liquids has been studied extensively for many years because of the importance of such data to the engineering and to the fundamental study of the liquid state. A number of investigators have studied pressure dependence for a variety of dense gases and liquids (*1* to *11*, *13*, *14*, *17* to *20*), which, coupled with simultaneous or independent investigations on the PVT behavior at high pressures, form the base of correlation procedures currently available.

During the course of this investigation, viscosity behavior of seven aliphatic and three aromatic aldehydes was established. The equipment was designed to provide for simultaneous measurement of fluid density. The details of the equipment, as well as the data obtained, have been presented elsewhere (*8*). The pressure range investigated was 0 to 20,000 lb./sq.in. gauge. The precision of pressure measurement was ±0.2%. The temperature was measured to an accuracy of 1/2°F. The viscosity values were estimated to be correct to within ±1%.

One method of correlating viscosity behavior of liquids with respect to pressure is due to Thodos and co-workers (*12*, *21*). Following the Abas-Zades (*12*) correlation procedure for thermal conductivities, Thodos et al. have developed similar correlations for residual viscosity. In this approach, residual viscosity $\mu - \mu_0$ is considered to be a function of the critical and physical parameters of a substance:

$$\mu - \mu_0 = \alpha\ T_c^a\ P_c^b\ v_c^c\ R^d\ M^e\ v^f \quad (1)$$

By dimensional analysis, the following reduced property correlation results:

$$(\mu - \mu_0)\ \xi = \beta\ Z_c^m\ \rho_R^n \quad (2)$$

For the data considered in the original Thodos' investigation, the correlation is shown by solid lines in Figure 1.

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